THE USE OF ELECTRICAL SOUNDING DATA TO ESTIMATE HYDRAULIC AQUIFER CHARACTERISTICS AT THE FIELD SCALE

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OUTLINE

- Assess an empirical relationship between the aquifer resistivity (ρ) and hydraulic conductivity (k) and/or transimissivity (K) using DC soundings.

- Discuss a simple equation of power-law relation between the resistivity (ρ) and transimissivity (K).

- Assess this relationship using published and measured data for different geological conditions.

HYDROGEOPHYSICAL BACKGROUND

HYDRAULIC CONDUCTIVITY

Combination of hydrogeological and geophysical data

- **Niwas and Singhal (1981)**
  $K = (k \cdot \sigma) \cdot T$
  and $K = (k \cdot \rho) \cdot S$
- **Yadav et al. (1993) and Yadav (1995)**
  $K = (k \cdot S^{-}) \cdot \rho^{-}$
- **Chandra et al. (2008)**
  $k = A \cdot \sigma$ and
  $K = A \cdot \sigma \cdot h$

Assumptions: $(k \cdot \rho)$, $(k \cdot \sigma)$ and $(k \cdot S^{-})$ are constant for homogeneous hydrogeological area.

Relationship of hydrogeological and geophysical data

- **Heigold (1979)**
  $k = 386.4 \, . \, \rho^{-0.923283}$
- **Kosiniski and Kelly (1981) and Frohlich et al. (1996)**
- **Purvance and Andricevic (2000)**
  $k = a \, . \, (\sigma')^{b}$

positive log k–log σ' for low S_{por} soil ($\sigma' \approx \sigma_{el}$) and a negative relation for high S_{por} ($\sigma' \approx \sigma_{int}$).

Theoretical petrophysical models

- **Mazak et al.(1994)**
- **Frohlich (1994)**
- **Khalil and Santos (2009)**

where, $\sigma^{-} = 1/\rho^{-} = \rho_w / \rho \cdot \rho^{-}_w$, $S^{-} = h/\rho^{-}$, $A = k \cdot \rho$, $S_{por} = 1/r = S_s \cdot \rho_s \cdot (1-\Phi)/\Phi$, r is the hydraulic radius, S_s is the specific surface area and ρ_s is the mineral density.

ANALYTICAL APPROACH

Darcy's &Ohm's laws,

$$q = -k \cdot \Delta h/L, \qquad J = -1/\rho \cdot \Delta V / L,$$

$$k = (q/\Delta h) \cdot (\Delta V/J) \cdot \sigma$$

Chandra et al. (2008):

$$k = A \cdot \sigma \quad \text{and} \quad K = A \cdot \sigma \cdot h$$

From these equations, a direct linear relationship between k and σ and/or ρ can be predicted.

First & second Archie's Laws,

$$F_i = a \cdot \Phi^{-m} \quad \text{and ,} \quad F_i = a \cdot \Phi^{-m} \cdot S_w^{-n}$$

Kozeny-Carmen Equation,

$$p = \Phi \; / a \, . \, (S_{por})^2 \cdot T$$

P: permeability, T: pore capillary tortuosity, a: shape factor and S_{por} is the inner surface area.

$$k = P \cdot \rho_w \cdot g / \mu$$

a power-law relation between ρ and k can be predicted, for similar water quality area.

$$k = A \cdot \rho^B \quad \text{and} \quad K = A \cdot \rho^B \cdot h$$

-*A and B: adjustable parameters using a least-squares fit between the observed and estimated k-values.*

-*We look for the value of free parameters that minimize the mean-square logarithmic error of fit,*

$$\varepsilon^2 = 1/N \sum [\, \ln k_e(i) - \ln k_o(i) \,]^2, \; \delta = e^{\varepsilon}$$ *δ is the error factor*

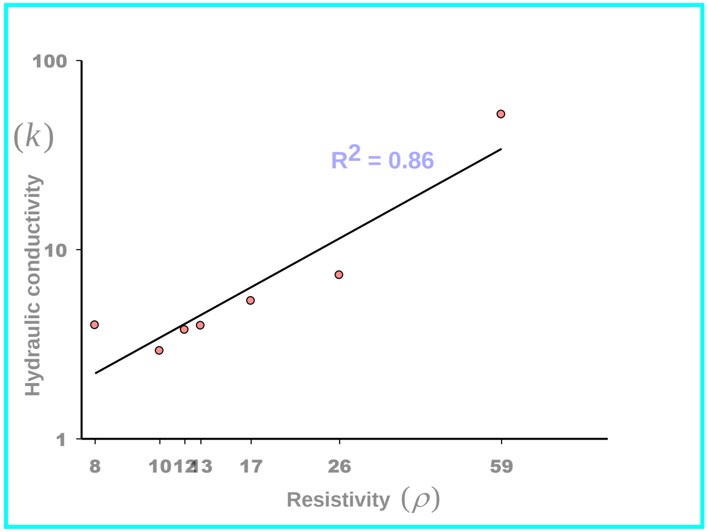
- *The positive and negative relationships will depend on the aquifer porosity and water saturation, as deduced from Archie's laws.*

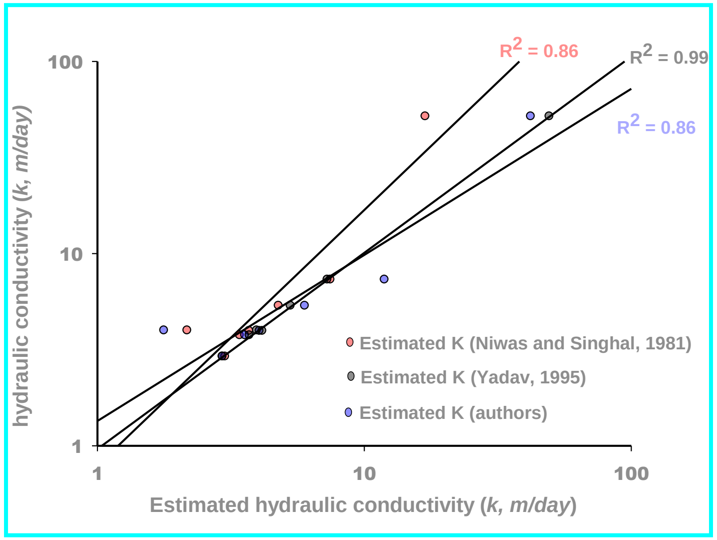
TESTING THE METHODOLOGY

Ex.1 Poor to moderate weathered sandstone aquifer of the Jayant aquifer, India (Yadav, 1995).

Sounding no. & well name	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	k-values (pumping test) m/day	$k = ((k \cdot \sigma) \cdot T)/h$, m/day	$k = ((k \cdot S^{-}) \cdot \rho^{-})/h = 18.14 \cdot \rho^{-}/h$ m/day	$k = A \cdot \rho^{B}$ m/day
1	41	7.5	3.97	89.2	3.96	1.8
2	62.7	11.8	3.7	214.6	3.7	3.6
3	77.7	12.8	3.9	288.4	4.2	4.1
4	53.9	10.4	2.9	162.6	2.9	2.9
5	44	16.5	5.3	210.5	5.3	5.9
6	48	25.8	7.3	359.1	7.3	11.9
7	30	58.5	51.7	508.95	49.4	42.1
				δ = 1.4	δ = 1	δ = 1.1

where $k{\cdot}\sigma = 0.29$, $A = 0.08$ *and* $B = 1.54$



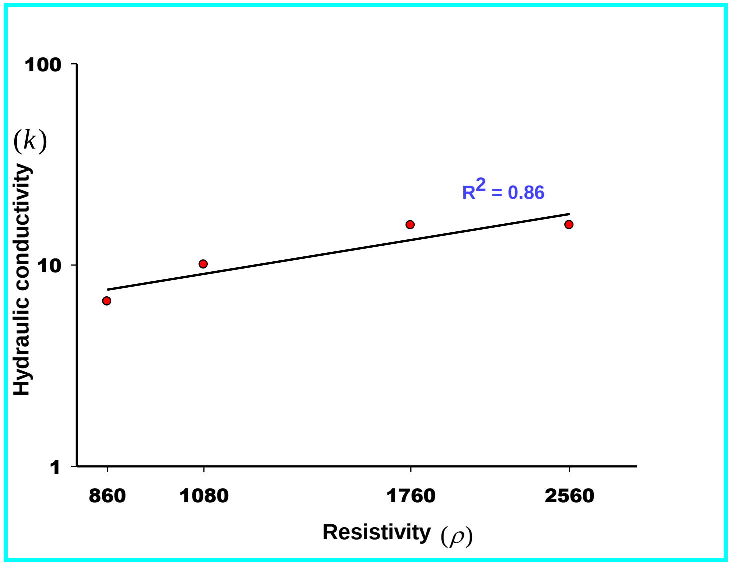


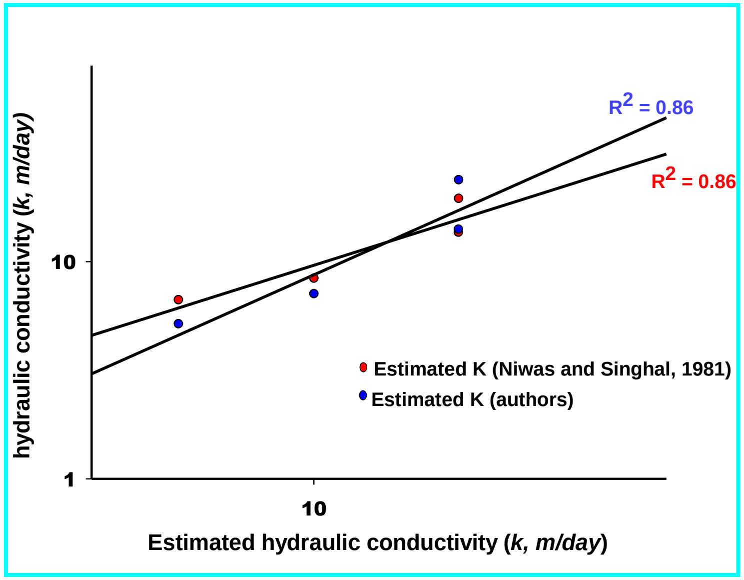
TESTING THE METHODOLOGY

Ex.2 White medium sand mixed with gravel aquifer, Nigeria, zone A, (Ekwe et al., 2006).

Sounding no.	Well name	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	k-values (pumping test) m/day	$k = ((K\cdot\sigma)\cdot T)/h$ m/day	$k = A\cdot\rho^B$ m/day
1	**Owerri**	62.7	1080	**10.02**	**8.3**	**7.1**
2	**Ihiagwa**	34.2	2560	**15.7**	**19.4**	**23.6**
3	**Egbu**	26.2	1760	**15.72**	**13.6**	**14**
4	**Ife**	8.3	860	**6.57**	**6.6**	**5.1**
					δ = 1	δ = 1.1

where $k\cdot\sigma = 0.0076$, $A = 0.0004$ *and* $B = 1.4$





TESTING THE METHODOLOGY

Ex.2 White medium sand mixed with gravel aquifer, Nigeria, zone A, (Ekwe et al., 2006).

Sounding no.	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	$k = ((k \cdot \sigma) \cdot T)/h$, (m/day)	$k = A \cdot \rho^B$, m/day
5	50.9	3120	**24.02**	**31.18**
6	51.9	3430	**26.41**	**35.6**
7	62.8	1054	**8.12**	**6.8**

where $k \cdot \sigma = 0.0076$, $A = 0.0004$ and $B = 1.4$

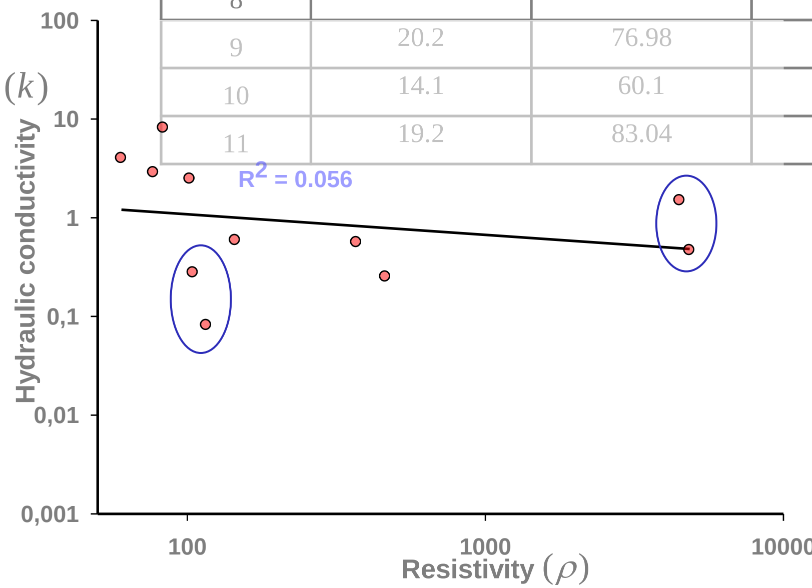
Aquifer parameters of sounding locations across zone A (Ekwe et al., 2006) and hydraulic conductivity estimation (k) .

TESTING THE METHODOLOGY

Ex.3 Hard rock granite aquifer, India, (Chandra et al., 2008).

Sounding no. & well name	Aquifer thickness h (m)	Aquifer resistivity ρ (Ωm)	Normalized aquifer resistivity ρ (Ωm)	k-values (pumping test) m/day	$k = ((k \cdot \rho^{-}) \cdot S^{-})/h = 188 \cdot S^{-}/h$ m/day	$k = A \cdot \rho^{B}$ m/day
1	20.8	115.9	217.3913	0.08	**0.87**	**1.4**
2	41.6	369.5	400	0.6	**0.46**	**0.6**
3	32	461.9	500	0.25	**0.38**	**0.5**
4	50	104.5	196.1	0.28	**0.96**	**1.5**
5	39	4850	200	0.47	**0.92**	**0.07**
6	25	144.8	149.3	0.59	**1.24**	**1.17**
7	36	4490.7	185.2	1.49	**1**	**0.075**
8	7	101.96	140.8	2.47	**1.36**	**1.56**
9	20.2	76.98	71.4	2.87	**2.6**	**1.95**
10	14.1	60.1	58.8	3.98	**3.1**	**2.38**
11	19.2	83.04	62.5	8.1	**2.96**	**1.84**

where $A = 63$ and $B = -0.8$ δ = 2.5 δ = 1.2



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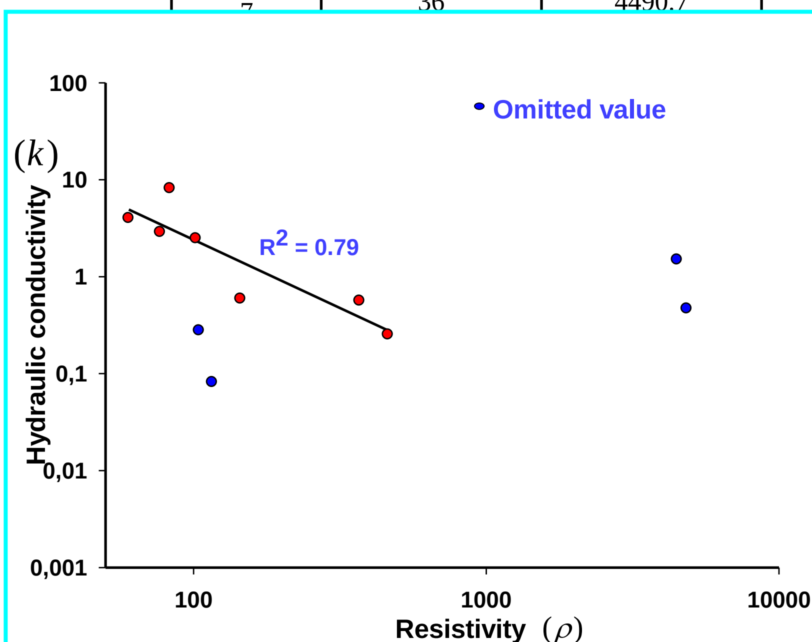
1	20.8	115.9	217.3913	0.08	0.87	
2	41.6	369.5	400	0.6	0.46	0.4
3	32	461.9	500	0.25	0.38	0.27
4	50	104.5	196.1	0.28	0.96	
5	39	4850	200	0.47	0.92	
6	25	144.8	149.3	0.59	1.24	1.23
7	36	4490.7	185.2	1.49	1	
			140.8	2.47	1.36	1.9
			71.4	2.87	2.6	2.8
			58.8	3.98	3.1	3.9
			62.5	8.1	2.96	2.5

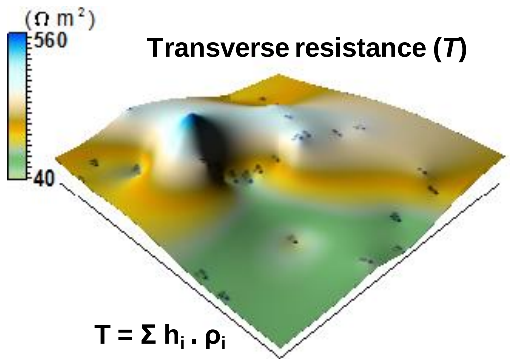
where A = 794 and B = -1.3 δ = 2.5 δ = 1.2



TESTING THE METHODOLOGY

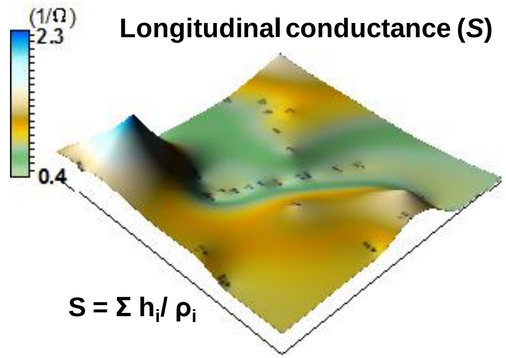
Ex.4 Holocene tidal deposits aquifer, Cuxhaven area.

Transverse resistance (*T*)

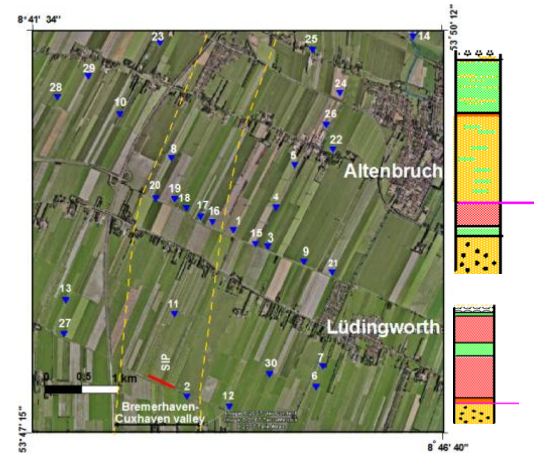


$T = \Sigma h_i . \rho_i$

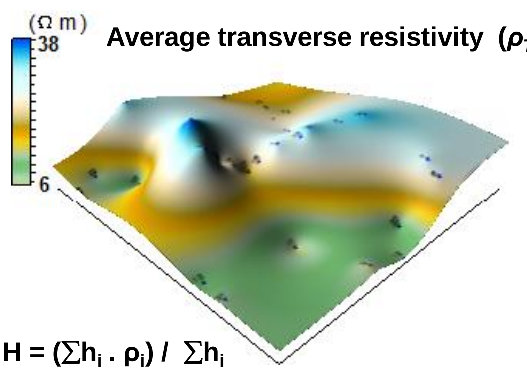
Longitudinal conductance (*S*)



$S = \Sigma h_i / \rho_i$

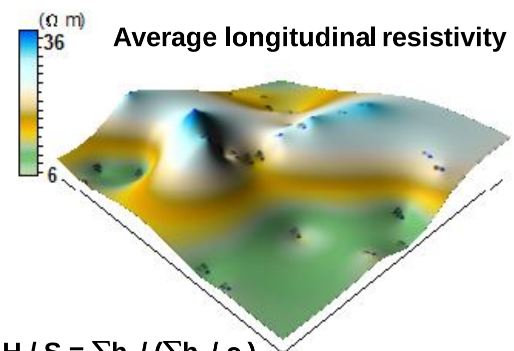


Average transverse resistivity (ρ_T)



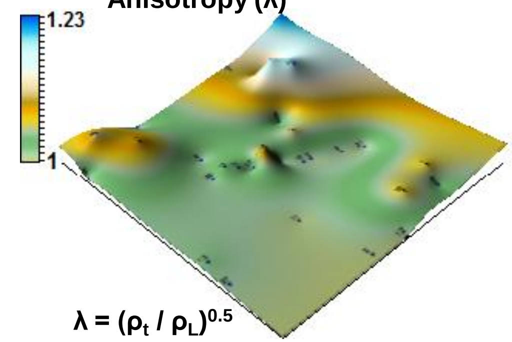
$\rho_t = T / H = (\sum h_i . \rho_i) / \sum h_i$

Average longitudinal resistivity (ρ_S)



$\rho_L = H / S = \sum h_i / (\sum h_i / \rho_i)$

Anisotropy (λ)



$\lambda = (\rho_t / \rho_L)^{0.5}$

- *The increase of T may reflect an increase in transmissivity (Niwas and Singhal, 1985).*
- *The increase of S may correspond to an average increase of clay content (Oteri, 1981).*
- *ρ_L is, in general, equal to ρ_t, which indicates that the current flow along the bedding planes is similar to that normal to the bedding planes (Flathe, 1955).*
- *λ is interpreted as a result of alternating layers of clay and fine sands and/or intercalation of different grain size in the same layer (Frohlich, 1974 and Singh, 1970).*

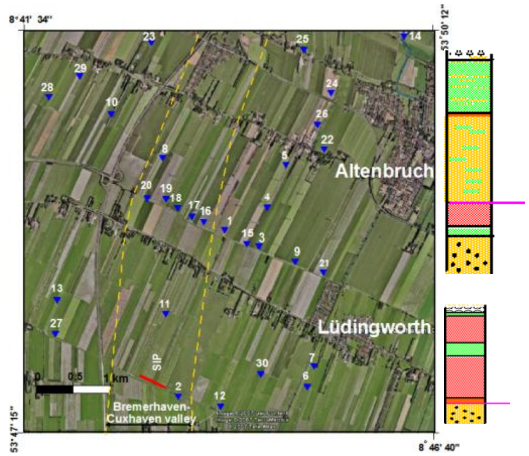
TESTING THE METHODOLOGY

Ex.4 Holocene tidal deposits aquifer, Cuxhaven area.

Sounding no.	Well name	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	k-values (grain size) m/day	*k* = $((K \cdot \sigma) \cdot T) / h$ m/day	*k* = $A \cdot \rho^{B}$ m/day
7	BEA01	7.92	32.3	**1.1**	**1**	**1.04**
20	BUC01	9.05	35.6	**1.3**	**0.7**	**1.2**
6	SUM01	3.17	32.3	**0.85**	**0.9**	**1.04**
14	BIN01	4.56	29.2	**0.86**	**0.86**	**0.87**

where $k \cdot \sigma = 0.029$, $A = 0.002$ *and* $B = 1.8$

$\delta = 1.1$ $\delta = 1$

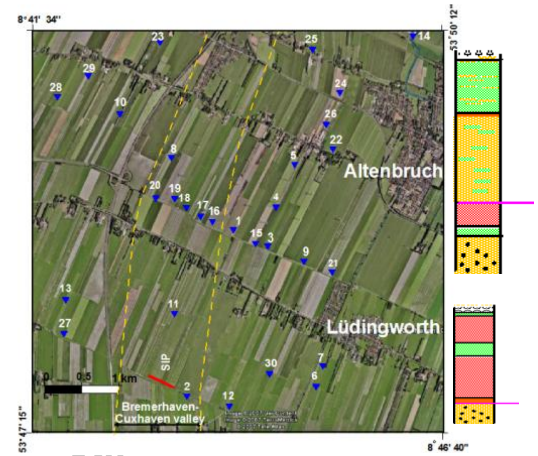


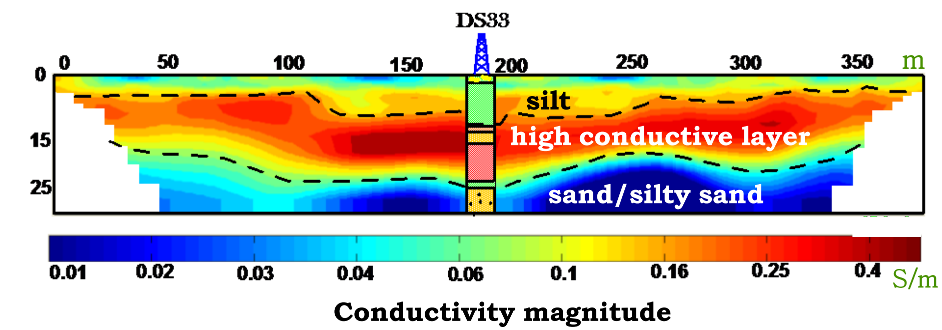
TESTING THE METHODOLOGY

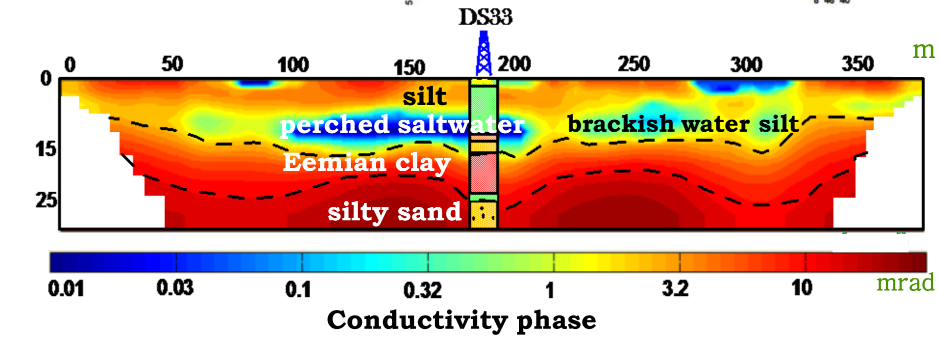
Ex.4 Holocene tidal deposits aquifer, Cuxhaven area.

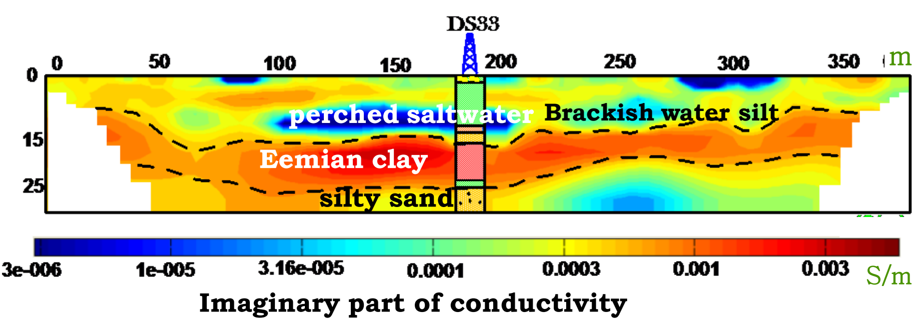
Sounding no.	Well name	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	k-values m/day	$k = ((K\cdot\sigma)\cdot T)/h$ m/day	$k = A\cdot\rho^B$ m/day
7	BEA01	7.92	32.3	1.1	1	1.04
20	BUC01	9.05	35.6	1.3	0.7	1.2
6	SUM01	3.17	32.3	0.85	0.9	1.04
14	BIN01	4.56	29.2	0.86	0.86	0.87

where $k\cdot\sigma = 0.029$, $A = 0.002$ and $B = 1.8$ — $\delta = 1.1$ — $\delta = 1$

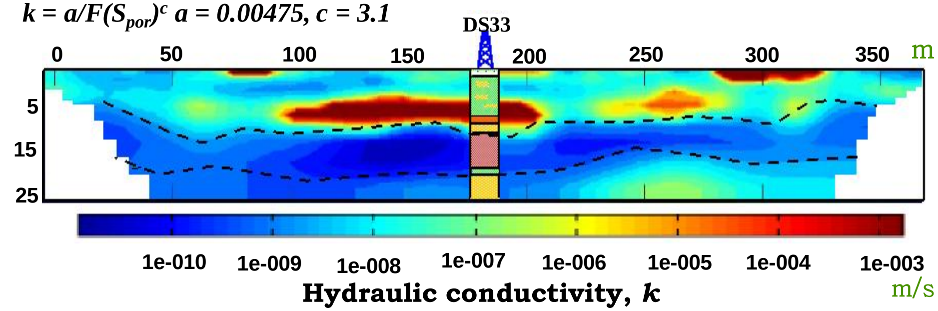








$k = a/F(S_{por})^c$ $a = 0.00475$, $c = 3.1$



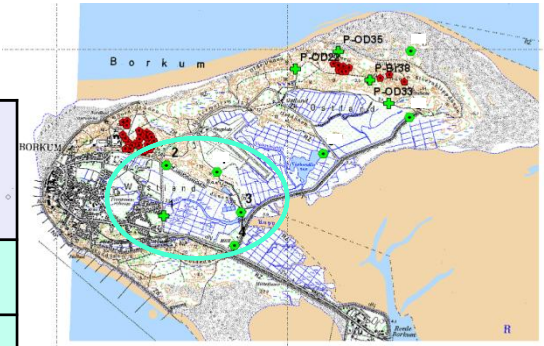
Frequency dependence of conductivity magnitude (top left), conductivity phase shift (top right), imaginary part of conductivity (below left) and hydraulic conductivity (below right) at 2.5 Hz.

TESTING THE METHODOLOGY

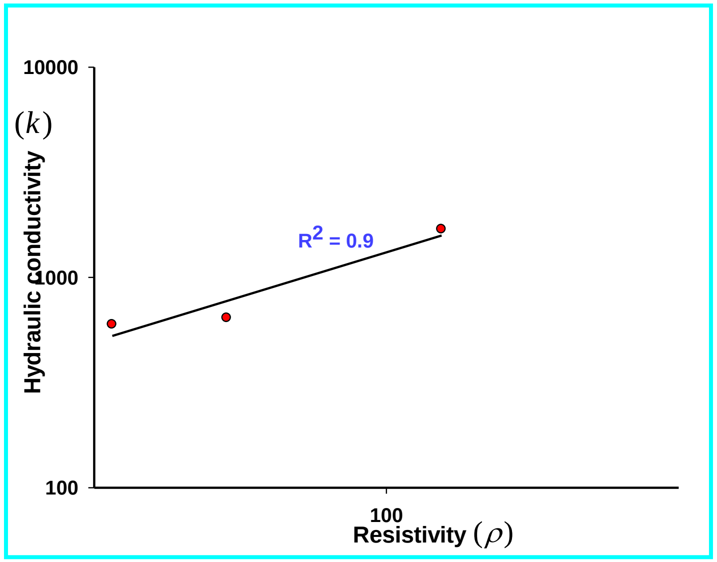
Ex.5 Holocene sandy aquifer, Borkum Westland.

Soundin g no.	Well name	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	k-values (pumping test) m/day	k = ((K·σ) · T) /h m/day	k = A · ρ^B m/day	k % error (±14% noise)
1	P-01	5.8	52.2	595.86	621.8	531.5	±16%
2	P-02	5.4	68.5	640	815.15	750.6	±21%
3	P-07	4.6	114	1690.44	1356.6	1433.3	±15%

where k·σ = 11.86, A = 3.5and B = 1.27 δ = 1 δ = 1



Average longitudinal resistivity values (ρ_L) were used as the aquifer resistivity values, where the current flow in saturated aquifer is essentially horizontal.



Soundin g no.	Aquifer thickness (h) m	Aquifer resistivity (ρ) Ωm	k = ((K·σ) · T) /h m/day	k = A · ρ^B m/day
4	3	51.7	615.23	525.05
5	5.88	37.7	448.63	351.58
6	3,23	66.9	796.11	728.38
7	4.26	74.3	832.1926	884.17

Aquifer parameters of sounding locations across Borkum Westland and hydraulic conductivity estimation (k) .

CONCLUSIONS

✓ For predicting the hydraulic conductivity (*k*) and transimissivity (*K*), a power-law relation between log ρ and log *k* has a reasonable degree of accuracy, using;

$$k = A \cdot \rho^{B} \quad and \quad K = A \cdot \rho^{B} \cdot h$$

✓ According to the present study:

1- For positive relation, the coefficient "B" ranges from 1.2 to 1.8. The positive power relation is expected for partially saturated or unsaturated unconsolidated aquifers ($S_w \neq 1$ and $\Theta \neq \Phi_{eff}$). The coefficient "*A*" is proportional to the rock matrix conductivity.

2- The negative power relation, i.e., *B* is negative, is expected for fully saturated clean sand and fractured consolidated aquifers ($S_w \neq 1$ and $\Theta \neq \Phi_{eff}$) e.g., hard rock and fully saturated sandy aquifer.

✓ For highly heterogeneous aquifers (e.g., dirty sand), this relation could not be effective, and so, SIP method will be better for predicting *k*, because of the implicit link with the pore/grain interface.

CONCLUSIONS